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Innovation The Research Concept

Gprw-closed sets in Bi-topological Spaces and their Properties

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Abstract

Objectives

The aim of this paper is to introduce and study the new class of sets called gprw closed sets in bi topological space.

Statistical Analysis

This new class of sets lies between the regular weakly closed(briefly rw closed) sets in bi topological space and the class of generalized pre regular closed (briefly gpr closed) sets.

Findings

This paper also aims to study the fundamental properties of the this class of sets in bi topological spaces.

Application/Improvements

The main application of this work is to fill the gap between the rw-closed sets and gpr-closed sets by this new class of sets in bi topological spaces.

Keywords: Gprw-Closed Sets, Bi-Topological Spaces, Topological Spaces, Generalization Of Closed Sets.

Introduction

Every topological space can be defined either with the help of axioms for the closed sets or the Kutatowiski closure axioms. So one can imagine that, how important the concept of closed sets is in the topological spaces. In 1970, levine[1]initiated the study of so-called generalized closed sets. By definition, a subset S of a topological space X is called generalized closed set if cl(A) ⊂ U whenever A ⊂ U and U is open. This notion has been studied extensively in the recent years by many topologists because generalized closed sets are not only natural generalization of closed sets. Moreover, they also suggest several new properties of topological spaces. Most of these new properties are separation axioms weaker than T_1, some of them found to be useful in computer science and digital topology. Furthermore, the study of generalized closed sets also provides new characterization of some known classes of spaces, for example, the class of extremely disconnected spaces by Caw, Ganster and Reilly[2]in 1997 Y. Gnanambal[3] proposed the definition of generalized preregular-closed sets (briefly gpr-closed) and further notion of preregular$T_{1/2}$ space and generalized preregular continuity was introduced. And in 2007, notion of regular weakly closed set is defined by S.S. Benchalli and R.S. Wali[4] and proved that this class lie between the class of all w-closed sets given by P. Sundaram and M. Sheik John[5] and the class of all regular generalized closed sets defined by N. Palaniappan and K.C. Rao[6].

Aim of the Study

In this paper, we introduced and studies new class of sets called generalized pre regular weakly closed set (briefly gprw-closed) in topological space which is properly placed between the regular weakly closed sets and generalized pre regular closed sets. K. Kannan[7] defined Soft Strongly g-Closed Sets. Topological Indices and Interpolation of Sequences are defined by M. Ebrahimi[8]. Generalization of Rough Topology given by A. Tripathi[9]

2 Preliminary Notes.

Definition 2.*1*

A subset $\mathbf{A}$ of $\mathbf{X}$ is called generalized closed ($\mathbf{briefly\ g-closed}$)1 set iff cl(A) ⊆ U whenever A ⊆ U and U is open.

Definition 2.2

A subset A of X is called regular open (briefly r-open)10 set if A = int(cl(A)) and regular closed (briefly r-closed)10 set if A = cl(int(A)).

Definition 2.3

A subset A of X is called pre-open set11 if A ⊆int(cl(A)) and pre-closed11 set if cl(int(A)) ⊆ A.

Definition 2.4

A subset A of X is called semi-open set12 if A ⊆ cl(int(A)) and semi-closed12 set if A ⊆int(cl(A)).

Definition 2.5

A subset A of X is called α-open13 if A ⊆int(cl(int(A))) and α-closed13 if cl(int(cl(A))) ⊆ A.

Definition 2.6

A subset A of X is called semi-preope14 if A ⊆cl(int(cl(A))) and semi-preclose14 if int(cl(int(A))) ⊆ A.

Definition 2.7

A subset A of X is called θ-closed15 if A = $\mathbf{cl}_{\boldsymbol{\theta}}$(A), where $\mathbf{cl}_{\boldsymbol{\theta}}$(A) = {x ∈X : cl(U) ∩ A = φ,U∈ τ andx∈U}.

Definition 2.8

A subset A of X is called δ-closed15 if A = $\mathbf{cl}_{\boldsymbol{\delta}}$(A), where$\mathbf{cl}_{\boldsymbol{\delta}}$(A)= {x ∈X :int(cl(U)) ∩ A = φ, U ∈ τ andx ∈ U}.

Definition 2.9

A subset A of a space (X, τ) is called regular semiopen16 if there is a regular open set U such that U ⊂ A ⊂ cl(U). The family of all regular semiopen sets of X is denoted by RSO(X).

Definition 2.10

A subset A of a space (X, τ) is said to be semi-regular open17 if it is both semiopen and semiclosed.

Definition 2.11

A subset of a topological space (X, τ) is called

1. Semi-generalized closed (briefly sg-closed)[18] if scl(A) ⊆ U whenever A ⊆ U and U is semiopen in X.
2. Generalized semiclosed (briefly gs-closed)[19] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
3. Generalized α-closed (briefly gα-closed)[20] if α − cl(A) ⊆ U whenever A ⊆ U and U is α-open in X.
4. Generalized semi-preclosed (briefly gsp-closed)[21] if spcl(A) ⊆ U whenever A ⊆ U and U is open in X.
5. Regular generalized closed (briefly rg-closed) [6] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
6. Generalized preclosed (briefly gp-closed)[22] if pcl(A) ⊆ U whenever A ⊆ U and U is open in X.
7. Generalized pre regular closed (briefly gpr-closed) [3] if pcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
8. θ-generalized closed (briefly θ − g-closed)[23]if $\mathbf{cl}_{\boldsymbol{\theta}}$(A) ⊆ U whenever A ⊆ U and U is open in X.
9. δ-generalized closed (briefly δ − g-closed)[24] if $\mathbf{cl}_{\boldsymbol{\delta}}$(A) ⊆ U whenever A ⊆ U and U is open in X.
10. Weakly generalized closed (briefly wg-closed)[26] if cl(int(A)) ⊆ U whenever A ⊆ U and U is open in X.
11. Strongly generalized closed[5] (briefly g∗-closed[27]) if cl(A) ⊆ U whenever A ⊆ U and U is g-open in X.
12. π-generalized closed (briefly π − g-closed) [25] if cl(A) ⊆ U whenever A ⊆ U and U is π-open in X.
13. Weakly closed (briefly w-closed)[26] if cl(A) ⊆ U whenever A ⊆ U and U is semiopen in X.
14. Mildly generalized closed (briefly mildly g-closed)[30] if cl(int(A)) ⊆ U whenever A ⊆ U and U is g-open in X.
15. Semi weakly generalized closed (briefly swg-closed)[27] if cl(int(A)) ⊆ U whenever A ⊆ U and U is semiopen in X.
16. Regular weakly generalized closed (briefly rwg-closed)[27] if cl(int(A)) ⊆ U whenever A ⊆ U and U is regular open in X.
17. Regular weakly closed (briefly rw-closed) [4] if cl(A) ⊆ U whenever A ⊆ U and U is regular semiopen in X.

The complements of the above mentioned closed sets are their respective open sets.

Theorem 2.12

Every regular semiopen set in X is semiopen but not conversely[30].

Theorem 2.13

If A is regular semiopen in X, then X/A is also regular semiopen[30].

Theorem 2.14

In a space X, the regular closed sets, regular open sets and clopensets are regular semiopen[30].

$\boldsymbol{gprw}$-Closed Sets in Bi-topological spaces

After investigating several closed sets through extensive study we now introduce to define the same class of that closed sets in Bi-topological spaces and also we will study the properties of our class and investigate that how they behaves under the new circumstances or conditions which we imposed on this class. So for this chapter the definition of $\boldsymbol{gprw}$-closed set in bi-topological spaces is as follows

Definition 3.1

Let $\boldsymbol{X}$ be a topological space with topology τ_1 and τ_2 . Then a set$\boldsymbol{A}$ said to be $(\mathbf{1},\mathbf{2})-\boldsymbol{gprw}$-closed in (X,τ_1,τ_2) if τ_2 $-\boldsymbol{pcl}(\boldsymbol{A}) \subset \boldsymbol{U}$, whenever $\boldsymbol{A} \subset \boldsymbol{U}$, where U is τ_1 −regular semi open.

Now with the help an example we find out, which subsets of the following topological space are $(\mathbf{1},\mathbf{2})-\boldsymbol{gprw}$-closed sets and which are not.

Example 3.2

Let $\boldsymbol{X} = \{\boldsymbol{a},\boldsymbol{b},\boldsymbol{c}\}$ be a bi-topological space with topologies $\tau_1 = \{\emptyset,\ \boldsymbol{X},\ \{\boldsymbol{a}\},\{\boldsymbol{b}\},\{\boldsymbol{a},\boldsymbol{b}\}\}$ $\tau_2 = \{\emptyset,\boldsymbol{X},\{\boldsymbol{b}\}\}$ then τ_1-regular semi open sets are $\{\emptyset,\ \boldsymbol{X},\{\boldsymbol{a}\},\{\boldsymbol{b}\},\{\boldsymbol{b},\boldsymbol{c}\},\{\boldsymbol{c},\boldsymbol{a}\}\}$ and τ_2 -pre-closed are $\{\emptyset,\ \boldsymbol{X},\{\boldsymbol{a}\},\{\boldsymbol{c}\},\{\boldsymbol{a},\boldsymbol{c}\}\}$, now let we take a set $\boldsymbol{A} = \{\boldsymbol{a}\}$ then τ_2 -$\boldsymbol{pcl}(\{\boldsymbol{a}\}) = \{\boldsymbol{a}\} \cap \{\boldsymbol{a},\boldsymbol{c}\} \cap \{\boldsymbol{X}\} = \{\boldsymbol{a}\}$, which is contained in τ_1-regular semi open set $\boldsymbol{U} = \{\boldsymbol{a}\}$. So $\boldsymbol{A}$ is $(\mathbf{1},\mathbf{2})-\boldsymbol{gprw}$ closed set. Similarly $\{\boldsymbol{c}\},\{\boldsymbol{a},\boldsymbol{b}\},\{\boldsymbol{a},\boldsymbol{c}\}$ all are $(\mathbf{1},\mathbf{2})-\boldsymbol{gprw}$ closed sets.

Now if we take $\boldsymbol{B} = \{\boldsymbol{b}\}$, then τ_2 -$\boldsymbol{pcl}(\{\boldsymbol{b}\}) = \boldsymbol{X}$, which is not contained in τ_1-regular semi open set $\{\boldsymbol{b}\}$, so it is not $(\mathbf{1},\mathbf{2})-\boldsymbol{gprw}$ closed set. Similarly $\{\boldsymbol{b},\boldsymbol{c}\}$ is also not $(\mathbf{1},\mathbf{2})-\boldsymbol{gprw}$ closed set.

Theorem 3.3

In a bi-topological space (X, τ_1, τ_2) any $(\mathbf{1,2}) - \boldsymbol{rw}$-closed set is $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set, but its converse is not true.

Proof: Let$A$ be an arbitrary$(1,2) - rw$-closed in (X, τ_1, τ_2) such that $A \subseteq U$ and U is τ_1 –regular semi open. By definition of $(1,2) - rw$-closed we have, $\tau_2 - cl(A) \subseteq U$. To prove A is $(1,2) - gprw$-closed it is sufficient to show that $\tau_2 - pcl(A) \subset U$. Since every closed set in a topological space is pre-closed [20] therefore $\tau_2 - pcl(A) \subseteq \tau_2 - cl(A)$, So we got $\tau_2 - pcl(A) \subseteq \tau_2 - cl(A) \subseteq U$, this shows that $\tau_2 - pcl(A) \subseteq U$, whenever $A \subseteq U$ and U is τ_1 –regular semi open. Hence A is $(1,2) - gprw$-closed set.

Converse of the above theorem is not true, that is if we take any $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set than it may not be $(\mathbf{1,2}) - \boldsymbol{rw}$-closed in a topological space it can be seen from the following example.

Example 3.4

In example 6.1.2 if we take a $\boldsymbol{A} = \{\boldsymbol{a}\}$ then τ_2 -$\boldsymbol{cl}(\{\boldsymbol{a}\}) = \{\boldsymbol{a}, \boldsymbol{c}\}$ which is not contained in a τ_1-regular semi open set $\{\boldsymbol{a}\}$ so it is not $(\mathbf{1,2}) - \boldsymbol{rw}$ closed, but it is $(\mathbf{1,2}) - \boldsymbol{gprw}$ closed as its τ_2 -$\boldsymbol{pcl}$ is contained in $\{\boldsymbol{a}\}$.

Theorem 3.5

In a topological space (X, τ_1, τ_2), any $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set is$(\mathbf{1,2}) - \boldsymbol{gpr}$-closed set, but its converse is not true.

Proof

Let (X, τ_1, τ_2) be a topological space and A be $(1,2) - gprw$-closed subset of X such that $A \subseteq U$ where U is τ_1 –regular open. Now we have the fact that every τ_1-regular open set is τ_1-regular semi open, therefore U is τ_1-regular semi open and by definition of $(1,2) - gprw$-closed set we have $\tau_2 - pcl(A) \subset U$ and as given that A contained in U. So we arrived at the stage at which $\tau_2 - pcl(A) \subset U$, whenever $A \subseteq U$ and U is τ_1-regular open that means$A$ is $(1,2) - gpr$-closed set in a topological space (X, τ).

Converse of the above theorem is not true. That is every $(\mathbf{1,2}) - \boldsymbol{gpr}$-closed set in topological space $\boldsymbol{X}$ is not $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set. It can be seen from the following example.

Example 3.6

Let $\boldsymbol{X} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$ be space with topologies $\tau_1 = \{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\}$ and $\tau_2 = \{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\}$In these topological spaces the τ_1-regular open sets are $\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}\}$ **and** τ_2 –pre closed sets are $\{\emptyset, \boldsymbol{X}, \{\boldsymbol{c}\}, \{\boldsymbol{d}\}, \{\boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}\}$. Now if we take $\boldsymbol{A} = \{\boldsymbol{a}, \boldsymbol{c}\}$ then its τ_2 -$\boldsymbol{pcl}$ will be $\{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}$ which is contained in τ_1-regular open set $\boldsymbol{X}$, so it is a $(\mathbf{1,2}) - \boldsymbol{gpr}$-closed, but it is not $(\mathbf{1,2}) - \boldsymbol{gprw}$ closed because we have a τ_1-regular semi open set $\{\boldsymbol{a}, \boldsymbol{c}\}$ which does not contains τ_2 -$\boldsymbol{pcl}$ of $\boldsymbol{A}$. Also we have some other subsets of $\boldsymbol{X}$ like $\{\boldsymbol{b}, \boldsymbol{c}\}$, $\{\boldsymbol{a}, \boldsymbol{d}\}$, $\{\boldsymbol{b}, \boldsymbol{d}\}$,which are also $(\mathbf{1,2}) - \boldsymbol{gpr}$-closed but not $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed sets in topological space $\boldsymbol{X}$. Hence from the following example we can see that not every$(\mathbf{1,2}) - \boldsymbol{gpr}$-closed set is $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed.

Corollary 3.7

In a bi-topological space(X, τ_1, τ_2), every closed set is$(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set. But its converse is not true.

Proof

Proof of this corollary follows from S.S Banchali and R.S Wali [3] and by theorem 6.1.3. Converse of this corollary is not true, that is every $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set is not always closed this can be seen from the following example.

Example 3.8

Let $\boldsymbol{X} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ be a bi-topological space with topologies $\tau_1 = \{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}\}$ $\tau_2 = \{\emptyset, \boldsymbol{X}, \{\boldsymbol{b}\}\}$ then τ_1-regular semi open sets are $\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{b}, \boldsymbol{c}\}, \{\boldsymbol{c}, \boldsymbol{a}\}\}$ and τ_2 -pre-closed are $\{\emptyset, \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{c}\}\}$, if we take $\boldsymbol{A} = \{\boldsymbol{a}, \boldsymbol{b}\}$ then it is $(\mathbf{1,2}) - \boldsymbol{gprw}$ closed but not closed.

Corollary 3.9

In a topological space (X, τ_1, τ_2), every regular closed set is $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set. But its converse is not true.

Proof

Proof of this corollary follows from S.S Banchali and R.S Wali [3] and by theorem 6.1.3. Its converse can be seen from the example 6.1.8 and the fact that every $(\mathbf{1,2})$ –regular closed is closed.

Corollary 3.10

In a topological space (X, τ_1, τ_2), every $\boldsymbol{\theta}$-closed set is $(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set. But its converse is not true.

Proof

Proof of this corollary follows from S.S Banchali and R.S Wali [3] and by theorem 6.1.3. Its converse can be seen from the example 6.1.8 and the fact that every $\boldsymbol{\theta}$-closed is closed.

Corollary 3.11

In a topological space(X, τ_1, τ_2), every $\boldsymbol{\delta}$-closed set set is$(\mathbf{1,2}) - \boldsymbol{gprw}$-closed set. But its converse is not true.

Proof

Proof of this corollary follows from S.S Banchali and R.S Wali [3] and by theorem 6.1.3. Its converse can be seen from the example 6.1.8 and the fact that every $\boldsymbol{\delta}$-closed is closed.

Corollary 3.12

In a topological space (X, τ_1, τ_2), every $\boldsymbol{\pi}$-closed set set is $(\mathbf{1},\ \mathbf{2}) - \boldsymbol{gprw}$-closed set. But its converse is not true.

Proof

Proof of this corollary follows from S.S Banchali and R.S Wali [3] and by theorem 6.1.3. Its converse can be seen from the example 6.1.8 and the fact that every $\boldsymbol{\pi}$-closed is closed.

Corollary 3.13

In a topological space (X, τ_1, τ_2), every $(\mathbf{1}, \mathbf{2}) - \boldsymbol{w}$-closed set is $(\mathbf{1},\ \mathbf{2}) - \boldsymbol{gprw}$-closed set. But its converse is not true.

Proof

Proof of this corollary follows from S.S Banchali and R.S Wali [3] and by theorem 6.1.3. Converse of this corollary can be seen from the example, let the same space and topology as given in example

Let $\boldsymbol{X} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}$ be space with topology $\tau_1 = \{\emptyset,\ \boldsymbol{X},\ \{\boldsymbol{a}\},\ \{\boldsymbol{b}\},\ \{\boldsymbol{a}, \boldsymbol{b}\},\ \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\}$and $\tau_2 = \{\emptyset,\ \boldsymbol{X},\ \{\boldsymbol{a}\},\ \{\boldsymbol{b}\},\ \{\boldsymbol{a}, \boldsymbol{b}\},\ \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}\}$. In this topological space the τ_2-pre-closed and τ_1-regular semi open are $\{\emptyset,\ \boldsymbol{X},\ \{\boldsymbol{c}\},\ \{\boldsymbol{d}\}, \{\boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\}, \{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}\}$, $\{\emptyset,\ \boldsymbol{X},\ \{\boldsymbol{a}\}, \{\boldsymbol{b}\},\ \{\boldsymbol{b}, \boldsymbol{c}\},\ \{\boldsymbol{a}, \boldsymbol{c}\},\ \{\boldsymbol{b}, \boldsymbol{d}\},\ \{\boldsymbol{a}, \boldsymbol{d}\},\ \{\boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d}\},\ \{\boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d}\}\}$ respectively Now if we take a subset $\{\boldsymbol{c}\}$ of $\boldsymbol{X}$ then it is $(\mathbf{1}, \mathbf{2}) - \boldsymbol{gprw}$-closed since we have τ_1-regular semi open set $\{\boldsymbol{b}, \boldsymbol{c}\}$ which contains $\{\boldsymbol{c}\}$ and also τ_2-$\boldsymbol{pcl}(\{\boldsymbol{c}\}) = \{\boldsymbol{c}\}$ contained in $\{\boldsymbol{b}, \boldsymbol{c}\}$. Now $\{\boldsymbol{c}\}$ is not $(\mathbf{1}, \mathbf{2}) - \boldsymbol{w}$-closed set because we have τ_1-semi open set $\{\boldsymbol{b}, \boldsymbol{c}\}$ containing $\{\boldsymbol{c}\}$, and $\boldsymbol{cl}(\{\boldsymbol{c}\}) = \{\boldsymbol{c}, \boldsymbol{d}\}$ does not contained in $\{\boldsymbol{b}, \boldsymbol{c}\}$.

Theorem 3.14

Every pre-closed set is $(\mathbf{1}, \mathbf{2}) - \boldsymbol{gprw}$-closed set in a topological space (X, τ_1, τ_2) but not conversely.

Proof

Let a subset $\boldsymbol{A}$ of (X, τ_1, τ_2) be pre-closed set, such that $\boldsymbol{A} \subset \boldsymbol{U}$, where $\boldsymbol{U}$ is τ_1–regular semi open set. Now we want to prove that our assumed set $\boldsymbol{A}$ is also $(\mathbf{1}, \mathbf{2}) - \boldsymbol{gprw}$-closed set in $\boldsymbol{X}$. As given $\boldsymbol{A}$ is pre-closed therefore $\tau_2 - \boldsymbol{pcl}(\boldsymbol{A}) = \boldsymbol{A}$. So we got $\tau_2 - \boldsymbol{pcl}(\boldsymbol{A}) \subset \boldsymbol{U}$ whenever $\boldsymbol{A} \subset \boldsymbol{U}$, and $\boldsymbol{U}$is τ_1–regular semi open set. Hence $\boldsymbol{A}$ is $(\mathbf{1}, \mathbf{2}) - \boldsymbol{gprw}$-closed in (X, τ_1, τ_2).

Converse of this theorem is not true, that is not every $(\mathbf{1}, \mathbf{2}) - \boldsymbol{gprw}$-closed set is pre-closed set. It can be seen from the following examples.

Example 3.15

Let $\boldsymbol{X} = \{\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}\}$ be a bi-topological space with topologies $\tau_1 = \{\emptyset,\ \boldsymbol{X},\ \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{a}, \boldsymbol{b}\}\}$ $\tau_2 = \{\emptyset, \boldsymbol{X}, \{\boldsymbol{b}\}\}$ then τ_1-regular semi open sets are $\{\emptyset,\ \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{b}\}, \{\boldsymbol{b}, \boldsymbol{c}\}, \{\boldsymbol{c}, \boldsymbol{a}\}\}$ and τ_2-pre-closed are $\{\emptyset,\ \boldsymbol{X}, \{\boldsymbol{a}\}, \{\boldsymbol{c}\}, \{\boldsymbol{a}, \boldsymbol{c}\}\}$, if we take $\boldsymbol{A} = \{\boldsymbol{a}, \boldsymbol{b}\}$ then it is $(\mathbf{1}, \mathbf{2}) - \boldsymbol{gprw}$ closed but not pre closed.

Conclusion

In this research article a new class of generalized closed sets is defined in bi-topological spaces is defined which actually exist between the regular weakly closed (briefly rw closed) sets in bi topological space and the class of generalized pre-regular closed (briefly gpr closed) sets. In this paper also some important properties of this class are also investigated.

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